

COURSE PROPOSAL: FIRST COURSE IN ABSTRACT ALGEBRA

1. Course Title: First Course in Abstract Algebra
2. Course Number: MTH2XX
3. Proposing Department: Mathematics and Statistics
4. Proposed by: Nandini Nilakantan, Preena Samuel
5. Units: 3-1-0-0 [11 credits]
6. Course Description:

(A) Objectives of the course: An introduction to group theory and ring theory.

(B) Content (number of lectures mentioned in brackets):

Group theory: Binary operation, and its properties, Definition of a group and examples. Subgroups, homomorphism of groups, Cosets, normal subgroups and quotient groups, correspondence theorem, isomorphism theorems. (3)

Groups as symmetries, isometries of the plane, finite and discrete groups of isometries, finite subgroups of the rotation group linear groups (classical groups, SU_2 , SO_3). (4)

Conjugacy classes, Lagrange's theorem, Actions of groups on sets, Cayley's theorem, orbit and stabilizers. (3)

Class equations, p-groups and Sylow's theorems.

Group automorphisms, Symmetric group and alternating group Automorphism groups of S_n , for $n > 2$: $Aut(S_n)$ is inner for all $n > 2$ and except 6, construction of an outer automorphism of S_6 . (4)

Direct and semi-direct products, isomorphism theorem for semi-direct products. (2)

Applications of sylow's theorem: simplicity of A_n where ($n > 4$). (1)

Classification of finite abelian groups and Finitely-generated abelian groups. (3)

Free groups, generators and relations, presentations of groups. E.g.: Dihedral groups. (2)

Nilpotent and solvable groups. Composition series, Jordan-Hölder theorem. (3)

Introduction to rings: definition, subrings, homomorphisms, ideals, quotient rings, correspondence theorem, isomorphism theorems. (3)

Characteristic of a ring, Chinese Remainder Theorem, Integral domains, field of fractions, prime and maximal ideals, polynomial rings. (3)

Unique factorization domain, Principal Ideal domain, Euclidean domain, Gauss' theorem, Irreducibility of Polynomials. (4)

Structure theorem of f.g. modules over a PID's, application to finitely generated abelian groups. (4)

(C) Pre-requisites: None

(D) Short summary: This is a compulsory course for MSc (MTH) students

7. Books/References:

(a) Reference materials: I. N. Herstein: Topics in Algebra, Wiley.

(b) J. Milne: Group Theory (Online).

(c) D. S. Dummit and R.M. Foote, Abstract Algebra, Wiley.

(d) M. Artin, Algebra, PHI.

(e) N. Jacobson, Basic Algebra I, Basic Algebra II, Dover Publications.

Date: 28 March 2024

Signature of the Proposer:

Signature of DPGC Convener Mathematics and Statistics

This course is approved/not approved

Chairman, SPGC